Sheet Coating by Drawdown Of Extruded Polymer

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INTRODUCTION

The many ways to coat a sheet have been capably reviewed by Booth,[1] Middleman,[2] Kistler,[3] Kistler and Scriven,[4] Pearson,[5] Ruschak,[6] Satas,[7] Tanner,[8] and Kistler and Schweizer.[9] When a sheet is coated by extrusion, a thin plastic coating is continuously applied to the flat substrate. In some extrusion coating processes, the sheet first contacts the melt inside the die. We call these pressure coating processes, one of which is illustrated in *Figure* 1. Otherwise, the coating is drawn freely before touching the sheet and such processes are variously called curtain coating, slot coating, slot orifice coating, or simply extrusion coating. *Figure* 2 illustrates one popular embodiment of curtain coating.

In curtain coating, molten plastic is extruded through a slit placed just above the moving substrate. Since the substrate speed exceeds the average extrudate velocity in the slit, the polymer is drawn down towards the substrate, producing a coating that can be far thinner than the slit. The speed ratio of the substrate (V_s) to the melt at the die exit (V_0) is called the drawdown ratio.

In pressure coating, the sheet motion superposes a drag flow on the pressure driven flow through the slit. Curtain coating eliminates this drag flow. So curtain coating subjects the melt to lower shear stresses in the die. Since high stresses can cause fracture, the high line speeds usually required by the sheet coating industry often make pressure coating inappropriate.

Because the shear stresses in the melt are lower for curtain coating than pressure coating, the drawing force is lower as well. This is especially important for coating a weak, delicate substrate such as paper. Curtain coating also eliminates the possibility of back flow through the guide slit, a problem arising in pressure coating with low viscosity melts.

Curtain coating presents some drawbacks. Instead of applying the molten plastic to the substrate under pressure, curtain coating draws the plastic onto the substrate, forming a weaker contact between the coating and the substrate than in pressure coating. Another disadvantage of curtain coating is that it can entrain air bubbles between the substrate and the coating.

An approximate analytic solution for coating a highly viscous Newtonian fluid onto a substrate is developed (low Poiseuille number and high capillary number). Two particular extrusion angles are considered: parallel and perpendicular to the moving substrate. We obtain expressions for the curtain shape, coating thickness, contact length, contact pressure, drawing force, apparent contact angle, and contact convexity. In this paper, we identify a process indeterminacy that arises in the curtain coating employing a parallel slit. We show that, by orienting the extrusion slit perpendicular to the moving substrate, this indeterminacy vanishes. Thus, a unique solution for the contact length is always obtained. The extension to a viscoelastic fluid is also briefly considered.

The critical region in curtain coating is the freely drawn film, where the melt undergoes large deformations and high stresses. The stresses are especially high at the contact point where the melt first touches the substrate. The coating thickness is determined in this freely drawn melt

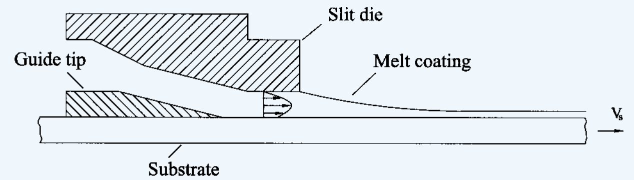

Figure 1—In pressure coating, the polymer melt contacts the substrate under pressure and inside the die.

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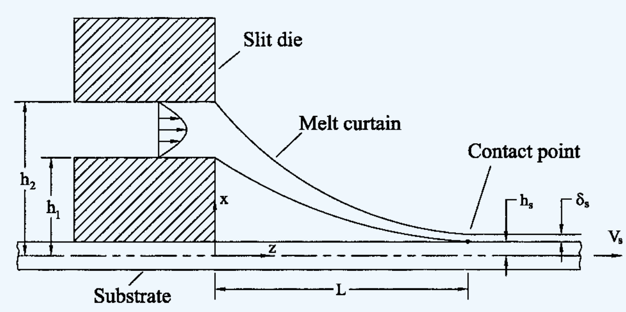

Figure 2—In parallel curtain coating, the melt curtain is extruded from a slit die and then drawn onto the substrate.

region. The stability of the freely drawn melt will govern the coating surface smoothness.

Scriven and Kistler[10] and Kistler and Scriven[11] carried out finite element simulations of curtain coating for a perpendicular extrusion angle. They include gravity, viscous forces, and surface tension. They showed that, for a fixed Reynolds number, both the contact length and the apparent contact angle θ_c increase as the drawdown ratio increases. For curtain coating, the Reynolds number is:

$$Re \equiv \frac{\rho \delta_s V_s}{\mu} \tag{1}$$

where V_s is the substrate speed, δ_s is the coating thickness, and μ is the fluid viscosity. For a constant drawdown ratio, they found that increasing the Reynolds number causes the contact line to shift downstream. In either case, the apparent contact angle eventually approaches 180°, which, they assert, is when catastrophic air entrainment occurs.

We believe that $\theta_c < 180°$, though desirable, is not necessary and that the contact convexity is equally important. Specifically, even the curtain contacting the substrate with $\theta_c = 180°$ may not entrain air when the contact convexity is high. High speed experiments by Blake and Ruschak[12] demonstrate curtain coating at an apparent contact angle of 180° without air entrainment. Of course, for presumably low contact convexity, many experiments confirm the 180° air entrainment angle in curtain coating[13] and in other coating geometries.[14,15] For a given contact angle, how much convexity is required to avoid the air entrainment will depend on the substrate roughness. We have not analyzed this dependence.

Regarding the apparent contact angle in curtain coating, for the perpendicular extrusion angle, Kistler and Scriven[11] calculated apparent contact angles dependent on both Reynolds number and the drawdown ratio, V_s/V_0. Specifically, for viscous curtains, they calculate the following scaling

$$Re \propto \left(\frac{V_s}{V_0}\right)^{0.3} \tag{2}$$

for fixed apparent contact angles exceeding 160°.

Dealy and Wissbrun[16] reviewed the literature on problematic phenomena associated with the extrusion of freely drawn plastic sheet. These rheology related phenomena include edge bead, neck-in, draw resonance, and web rupture. Our analysis neglects these phenomena.

Table 1 lists previous studies on curtain coating. Most focus on low viscosity fluids, where gravity, fluid inertia, and surface tension matter (and where viscous forces do not). These flows are called inviscid. Some analyses include pressure applied to the top of the curtain. *Table* 1 also lists analyses of stability upon two and three dimensional flow disturbances, the common line stability, and the critical conditions for the air entrainment. Some curtain coating experiments under nearly inviscid conditions are also listed in *Table* 1.

At low drawdown ratios, inviscid flow analyses normally predict that material near the inner free surface first moves upstream of the contact line, forming a fluid heel nearby (negative contact length). Blake et al.[17] studied the heel and, in this context, studied the effects of the substrate speed, curtain impingement speed, and impingement angle on the contact length. For an inviscid curtain forming a heel, they derived an accurate analytical solution for the contact length.

This paper provides an approximate analytical solution for coating highly viscous polymer melts ($Re<<1$) and gives expressions for the shape of the freely drawn melt curtain, the coating thickness, the contact length, the apparent contact angle, the contact convexity, the contact pressure, and the drawing force. Our objective is to determine these quantities, given the volumetric flow rate of the polymer melt, the die dimensions, the coating width, and the substrate speed. We further explore the role played by the drawing force. Here we are specifically concerned with the case having neither a vacuum applied inside the curtain nor air pressure applied outside.[13,18]

There are two popular slit orientations in curtain coating. The slit can be either parallel or perpendicular to the moving substrate. Both cases are analyzed here. When an intermediate extrusion angle is used, the process is called angular plane curtain coating.[1] These are used to coat weak webs. Using the same approach employed in this paper, we previously analyzed angular plane curtain coating.[19] This paper provides detailed explicit solutions for the two popular slit orientations.

SIMPLIFICATIONS

1. The polymer melt can be represented as an incompressible fluid. We will examine two cases: a Newtonian fluid and a Noll simple fluid.[20-23]

2. For commercial high speed melt coating, Re<<1. Thus fluid inertia is negligible.

3. For commercial high speed melt coating, the Poiseuille number

$$Ca \equiv \frac{\mu V_0}{\sigma_s} >> 1 \tag{3}$$

where g is the acceleration due to gravity, h_1 is the distance between the substrate midplane and the die exit (*Figure* 2) and L is the contact length. Thus, gravity can be ignored.

4. Neglect exit effects immediately downstream of the die, such as extrudate swell, which matter in viscoelastic fluids. Hence the position $z = 0$ is chosen at the die exit.

5. Assume that the melt velocity V_0 is position independent across the die orifice.

6. For commercial high speed melt coating, the capillary number

$$Ca \equiv \frac{\mu V_0}{\sigma_s} >> 1 \tag{4}$$

where σ_s is the surface tension. Hence, surface tension effects can be neglected with respect to viscous effects in the momentum balance.

7. Since the contact length L is well below the coating width w, we can further neglect the melt motion in the transverse direction, thus $v_y = 0$ everywhere in the curtain.

8. Assume that the pressure on both sides of the melt curtain is the atmospheric pressure.

ANALYSIS: PARALLEL SLIT

Figure 2 locates our rectangular coordinate system for the case where the extrusion slit is oriented parallel to the moving substrate. At $z = 0$:

$$\begin{aligned} x^i &= h_1 \\ x^o &= h_2 \\ v_z &= V_0 \equiv \frac{Q}{w(h_2 - h_1)} \end{aligned} \tag{5}$$

where w is the coating width, and x^i, x^0, are the x-coordinates of particles on the inner and outer curtain surfaces respectively. In this paper, the superscript i (or o) denote the quantities on the inner (or outer) curtain surfaces.

We seek a solution in the form of an extensional flow, a subclass of potential flows, such that in rectangular coordinates, the potential function is

$$\Phi = -\frac{1}{2}\left(a_1x^2 + a_2y^2 + a_3z^2\right) - b_1x - b_2y - b_3z \tag{6}$$

The corresponding velocity distribution is

$$\begin{aligned} v_x &= a_1x+b_1 \\ v_y &= a_2y+b_2 \\ v_z &= a_3z+b_3 \end{aligned} \tag{7}$$

Continuity requires

$$a_1 + a_2 + a_3 = 0 \tag{8}$$

From equation (5),

$$b_3 = V_0 \tag{9}$$

and by simplification 7,

$$a_2 = b_2 = 0 \tag{10}$$

In view of equations (8) and (10),

$$a_1 + a_3 = 0 \tag{11}$$

Thus

$$a_1 = -a_3 \tag{12}$$

here we introduce

$$a \equiv a_3 \tag{13}$$

which we call the curtain shape factor. From the boundary condition $v_x = 0$ at $x = 0$, we have

$$b_1 = 0 \tag{14}$$

In summary,

$$\begin{aligned} v_x &= -ax \\ v_y &= 0 \\ v_z &= az+V_0 \end{aligned} \tag{15}$$

Table 1—Summary of Curtain Coating Studies

	Analytical Solution	Numerical Solution	Experiment	Inviscid Flow	Viscous Flow	0 (*Re*)	Gravity	Surface Tension	Flow Distrubance and Stability Analysis	Pressure Difference Across the Curtain Thickness	Apparent Contact Angle	Air Entrainment	Angular Coating
1. Kistler[3] 2. Scriven and Kistler[10] 3. Kistler and Scriven[11] 4. Kistler and Scriven[4]		X			X	0.1~10	X	X			X	X	
5. Finnicum et al.[26]	X		X	X		10	X	X	X	X			X
6. Blake et al.[17]	X		X	X		10	X	X			X	X	
7. Clarke[27]			X	X		10					X	X	
8. Wienstein et al.[28,29]	X		X	X		10	X	X	X	X			
9. Wienstein et al.[30]	X			X		>>1	X	X	X	X			
10.Benkreira and Cohu[31]			X	X		10					X	X	X
11. Ding et al.[19]	X				X	<<1					X		X
12. This paper	X				X	<<1					X		

We now seek the pathline. Integrating equation (15) with initial conditions at $t = 0$,

$$x = x_0,\ y = y_0,\ z = z_0 = 0 \tag{16}$$

gives

$$\frac{x}{x_0} = e^{-at} \qquad \frac{y}{y_0} = 1 \qquad \frac{az + V_0}{V_0} = e^{at} \tag{17}$$

Eliminating t gives the shapes of both curtain surfaces (*Figure* 3),

$$\frac{x}{x_0} = \left(\frac{az}{V_0} + 1\right)^{-1} \tag{18}$$

For the inner curtain surface, $x_0^i = h_1$ and $z_0^i = 0$, and for the outside surface, $x_0^o = h_2$ and $z_0^o = 0$. Since the curtain is thin, from equation (18), we obtain the components of unit normals on both curtain surfaces.

$$\begin{aligned} \xi_x^o &\approx -\xi_x^i = \left[1 + \left(\frac{ah_1}{V_0}\right)^2 \left(\frac{x^i}{h_1}\right)^4\right]^{-1/2} \\ \xi_y^o &= \xi_y^i = 0 \\ \xi_z^o &\approx -\xi_z^i = \left(\frac{ah_1}{V_0}\right)\left(\frac{x^i}{h_1}\right)^2 \left[1 + \left(\frac{ah_1}{V_0}\right)^2 \left(\frac{x^i}{h_1}\right)^4\right]^{-1/2} \end{aligned} \tag{19}$$

Incompressible Newtonian Fluid

From equation (15), for an incompressible Newtonian fluid, the equation of motion (recall that fluid inertia is neglected) reduces to

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \tag{20}$$

which means that, p = constant in the curtain.

On both the inner and outer curtain surfaces, the x-component of the momentum balance requires that

$$(T_{xx} + p_0)\ \xi_x = 0 \tag{21}$$

or that

$$(-p + 2\mu a + p_0)\ \xi_x = 0 \tag{22}$$

In agreement with equation (20),

$$p = p_0 + 2\mu a \tag{23}$$

The y-component of the momentum balance is satisfied identically on both curtain surfaces. The z-component, on both surfaces, takes a form

$$(-p - 2\mu a + p_0)\ \xi_z = 0 \tag{24}$$

Equations (21-24) are valid only when $\delta_x >> \delta_z$, or inserting equation (19), when

$$\frac{ah_1}{V_0} << 1 \tag{25}$$

where ah_1 is the magnitude of v_x at the die exit.

The z-component of the force that the substrate exerts on the liquid coating (beyond the opposing force of atmospheric pressure) is,

$$\begin{aligned} F_z &= \int_0^w \int_{h_s}^{h_s+\delta_s} (-T_{zz} - p_0)\, dx dy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (p - S_{zz} - p_0)\, dx dy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (S_{xx} - S_{zz})\, dx dy \\ &= (S_{xx} - S_{zz})\, w\delta_s \end{aligned} \tag{26}$$

As expected, F_z is positive. From equation (26),

$$a = \frac{F_z}{4\mu w_s} \tag{27}$$

Assuming a constant density, the overall mass balance requires a coating thickness of

$$\delta_s = \frac{V_0(h_2 - h_1)}{V_s} \tag{28}$$

Evaluating equation (18) at the contact point ($x^i = h_s$, $z^i = L$) yields

$$L = \frac{V_0}{a}\left[\left(\frac{h_1}{h_s}\right) - 1\right] \tag{29}$$

Finally, from the z-component of the velocity in equation (15)

$$V_s = aL + V_0 \tag{30}$$

Given the drawing force F_z, volumetric flow rate of the polymer melt Q [thus V_0 from equation (5)], coating width w, die dimensions (h_1, h_2) and substrate half thickness h_s, we can first determine the product aL from equation (29), then the substrate speed V_s from equation (30), the coating thickness δ_s from equation (28), the curtain shape factor a from equation (27), the contact length L from equation (29) or (30), and finally the curtain shape from equation (18).

Interestingly, if one specifies V_s rather than F_z, the problem may become indeterminate. By indeterminate, we mean that equations (29) and (30) result in two independent equations for the product aL.

Inserting equation (27) into equation (29) gives

$$L = \frac{4V_0\mu w\delta_s}{F_z}\left[\left(\frac{h_1}{h_s}\right) - 1\right] \tag{31}$$

In terms of the dimensionless contact length $L^* \equiv L/h_s$ and the dimensionless drawing force $F_z^* \equiv F_z/4\mu w V_0$, equation (31) reduces to

$$L^* F_z^* = \delta_s^*(h_1^* - 1) \tag{32}$$

where $\delta_s^* \equiv \delta_s/h_s$ is the dimensionless coating thickness, and $h_1^* \equiv h_1/h_s$ is the dimensionless distance between the slit and the substrate. Equation (32) is a main result of this paper.

We next seek the apparent contact angle defined, by convention, as

$$\theta_c \equiv \pi - arctan\left(-\left.\frac{\partial x^i}{\partial z^i}\right|_{z^i=L}\right) \tag{33}$$

Table 2—Slit Die Dimensions and Coating Parameters

$h_2 \times 10^3$ (m)	$h_2 \times 10^3$ (m)	a (s^{-1})	V_0(m/s)	F_z (N)
2.500	3.000	20.00	1.000	72.00
5.000	6.000	22.50	0.5000	81.00
7.500	9.000	23.33	0.3333	83.99

Differentiating equation (18), then substituting with equations (27) and (29), gives

$$\left.\frac{\partial x^i}{\partial z^i}\right|_{z^i=L} = \frac{-F_z h_s^{\,2}}{4\mu w \delta_s V_0 h_1} = \frac{-F_z^*}{\delta_s^* h_1^*} \tag{34a}$$

Hence

$$\theta_c = \pi - \arctan\left(\frac{F_z^*}{\delta_s^* h_1^*}\right) \tag{34b}$$

In practice, when the apparent contact angle approaches 180°, air is entrained between the coating and the substrate, and adhesion is lost. When the apparent contact angle is large, we believe that entrainment can still be avoided if the contact convexity is high. We can obtain the contact convexity by differentiating equation (18) twice,

$$\left.\frac{\partial^2 x^i}{\partial {z^i}^2}\right|_{z^i=L} = \frac{2}{h_1}\left(\frac{a h_1}{V_0}\right)^2\left(\frac{V_0}{V_s}\right)^3 \tag{35}$$

Example 1: Die Design

Given the substrate half thickness $h_s = 5.000 \times 10^{-4}$ m, coating thickness $\delta_s = 1.000 \times 10^{-4}$ m, coating width $w = 0.9000$ m, substrate speed $V_s = 5.000$ m/s, melt viscosity $\mu = 1.000 \times 10^4$ Pa·s and contact length $L = 0.2000$ m, find appropriate pairs of die dimensions (h_1, h_2), along with corresponding drawing forces F_z.

Equations (27-30) give us four equations in five unknowns: a, F_z, h_1, h_2, V_0. *Table* 2 lists some combinations.

Incompressible Noll Simple Fluid

Nearly all viscoelastic fluid behavior falls into a general class of rheological models called the Noll simple fluid. For an incompressible Noll simple fluid, the extra stress tensor ***S*** is a function of the history of either the right relative Cauchy-Green tensor[20,22] or simply the relative deformation gradient.[21] Given the particle paths in equation (18), both the right relative Cauchy-Green strain tensor and the relative deformation gradient are independent of position.[24.25] This means that the components of ***S*** for a Noll simple fluid are also independent of position[24,25] except for the dependence on temperature of the physical parameters used to describe a particular member of this class of fluids.

The equation of motion reduces to equation (20) with the same arguments used in the Newtonian analysis. We conclude that

$$p = p_0 + S_{xx} \tag{36}$$

and that three components of the momentum balance are satisfied. Hence,

$$\begin{aligned} F_z &= \int_0^w \int_{h_s}^{h_s+\delta_s} (-T_{zz} - p_0)\, dxdy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (p - S_{zz} - p_0)\, dxdy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (S_{xx} - S_{zz})\, dxdy \\ &= (S_{xx} - S_{zz})\, w\delta_s \end{aligned} \tag{37}$$

In view of equation (26)

$$(S_{xx} - S_{zz}) \sim a \tag{38}$$

but the proportionality factor depends upon the particular type of Noll simple fluid, and this factor must therefore be measured in an extensional flow. Thus, for viscoelastic fluids, the parameter *a* must be viewed as unknown. Equations (28-30) still apply, but they are not sufficient to specify L and V_s.

ANALYSIS: PERPENDICULAR SLIT

Figure 4 locates our coordinate system for the case where the extrusion slit is oriented perpendicular to the moving substrate. At $x = d$:

$$z^i = 0,\ z^0 = \delta_0 \tag{39}$$

and

$$\upsilon_x = -V_0 = -\frac{Q}{w\delta_0} \tag{40}$$

We will seek a solution in the form of equation (6) with the corresponding velocity distribution in equation (7). Continuity again requires equation (8). Following the analysis for the parallel slit, we still obtain equations (11) and (12). Since $v_x = 0$ at $x = 0$, we again obtain equation (14). Further, from the boundary condition equation (40), one gets

$$a_3 = -a_1 = a\, V_0/d \tag{41}$$

In summary,

$$\begin{aligned} v_x &= -a_x \\ v_y &= 0 \\ v_z &= a_z + b_3 \end{aligned} \tag{42}$$

where the curtain shape factor *a* is given by equation (41).

We now derive the pathline. Integrating equation (42) with initial conditions at $t = 0$,

$$x = x_0 = d,\ y = y_0,\ z = z_0 \tag{43}$$

gives

$$\begin{aligned} \frac{x}{d} &= e^{-at} \\ \frac{y}{y_0} &= 1 \\ \frac{az + b_3}{az_0 + b_3} &= e^{at} \end{aligned} \tag{44}$$

Table 3—Comparison of Results for Parallel and Perpendicular Slit Curtain Coating

	Parallel Slit	Perpendicular Slit
Inner Curtain Shape	$\frac{x^i}{h_1}=\left(\frac{az^i}{V_0}+1\right)^{-1}$	$\frac{x^i}{d}=\left(\frac{az^i}{b_3}+1\right)^{-1}$
Outer Curtain Shape	$\frac{x^o}{h_2}=\left(\frac{az^o}{V_0}+1\right)^{-1}$	$\frac{x^o}{d}=\left(\frac{az^o+b_3}{a\delta_0+b_3}\right)^{-1}$
Curtain Shape Factor	$a=\frac{F_z}{4\mu w\delta_s}$	$a=\frac{F_z}{4\mu w\delta_s}=\frac{V_0}{d}$
Coating Thickness	$\delta_s=\frac{Q}{V_s w}=\frac{V_0(h_2-h_1)}{V_s}$	$\delta_s=\frac{Q}{V_s w}=\frac{V_0\delta_0}{V_s}$
Contact Length	$L=\frac{V_0}{a}\left[\left(\frac{h_1}{h_s}\right)-1\right]$	$L=\frac{b_3}{a}\left[\left(\frac{d}{h_s}\right)-1\right]$
Contact Pressure	$p=p_0+2\mu a$	$p=p_0+2\mu a$
Drawing Force	$F_z=4\mu a w\delta_s$	$F_z=4\mu a w\delta_s$
Apparent Contact Angle	$\theta_c=\pi-arctan\left(\frac{F_z^*}{\delta_s^* h_1^*}\right)$	$\theta_c=\pi-arctan\left(\frac{F_z^*}{\delta_s^* d^*}\right)$
Contact Convexity	$\left.\frac{\partial^2 x^i}{\partial z^{i2}}\right\|_{z^i=L}=\frac{2}{h_1}\left(\frac{ah_1}{V_0}\right)^2\left(\frac{V_0}{V_s}\right)^3$	$\left.\frac{\partial^2 x^i}{\partial z^{i2}}\right\|_{z^i=L}=\frac{2}{d}\left(\frac{ad}{b_3}\right)^2\left(\frac{b_3}{V_s}\right)^3$

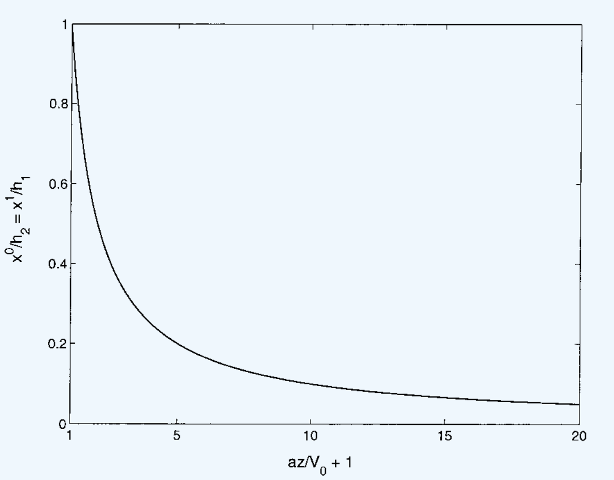

Figure 3—The shape of inner and outer curtain surfaces in parallel curtain coating, given by equation (18). The shapes of inner and outer surfaces match.

Eliminating t gives the curtain shape,

$$\frac{x}{d}=\left(\frac{az+b_3}{az_0+b_3}\right)^{-1} \tag{45}$$

For the inner curtain surface, $x_0^i=d$ and $z_0^i=d$, and for the outer surface, $x_0^o=d$ and $z_0^o=\delta_0$. From equation (45), the components of unit normals on both curtain surfaces are,

$$\begin{aligned}
\xi_x^o &\approx -\xi_x^i=\left[1+\left(\frac{ad}{b_3}\right)^2\left(\frac{x^i}{d}\right)^4\right]^{-1/2}\\
\xi_y^o &= \xi_y^i = 0\\
\xi_z^o &\approx -\xi_z^i=\left(\frac{ad}{b_3}\right)\left(\frac{x^i}{d}\right)^2\left[1+\left(\frac{ad}{b_3}\right)^2\left(\frac{x^i}{d}\right)^4\right]^{-1/2}
\end{aligned} \tag{46}$$

Incompressible Newtonian Fluid

From equation (42), for an incompressible Newtonian fluid, the equation of motion reduces to equation (20), thus

p = constant in the curtain. Again, this is valid only when $\delta_x >> \delta_z$, or inserting equation (46), when:

$$\frac{V_0}{b_3} << 1 \tag{47}$$

where b_3 (v_z at the slit) is a dependent variable.

Three components of the momentum balance are the same as those in the parallel slit. Equation (26) again gives the z-component of the force that the substrate exerts on the liquid coating (beyond the opposing force of atmospheric pressure). The coating thickness is given by

$$\delta_s = \frac{V_0 \delta_0}{V_s} \tag{48}$$

Inserting equation (41) into equation (26), one gets

$$\frac{V_0}{d} = \frac{F_z}{4\mu w \delta_s} \tag{49}$$

Evaluating equation (45) at the contact point ($x^i = h_s$, $z^i = L$), and combining with equation (41) gives

$$L = \frac{b_3 d}{V_0}\left[\left(\frac{d}{h_s}\right) - 1\right] \tag{50}$$

Finally from the z-component of the velocity in equation (42)

$$V_s = \left(\frac{V_0}{d}\right)L + b_3 \tag{51}$$

Given the drawing force F_z (or the substrate speed V_s), volumetric flow rate of the polymer melt Q [thus V_0 from equation (40)], coating width w, die dimension δ_0, distance d and substrate half thickness h_s, one can combine equations (48-51) to determine δ_s, V_s (or F_z), b_3 and L. Thus, the indeterminacy arising in the parallel slit curtain coating is eliminated.

Combining equations (47) and (51), we find that these solutions apply when:

$$\frac{1}{\frac{V_s}{V_0} - \frac{L}{d}} << 1 \tag{52}$$

Inserting equation (49) into (50), and then introducing the dimensionless contact length $L^* \equiv L/h$ and the dimensionless drawing force $F_z^* \equiv F_z/4\mu w b_3$ yields

$$L^* F_z^* = \delta_s^* (d^* - 1) \tag{53}$$

where $\delta_s^* \equiv \delta_s/h_s$ is the dimensionless coating thickness, and $d^* \equiv d/h_s$ is the dimensionless distance between the slit and substrate.

We now seek the apparent contact angle defined by equation (33). Differentiating equation (45), and combining with equations (49) and(50) gives

$$\theta_c = \pi - \arctan\left(\frac{F_z^*}{\delta_s^* d^*}\right) \tag{54}$$

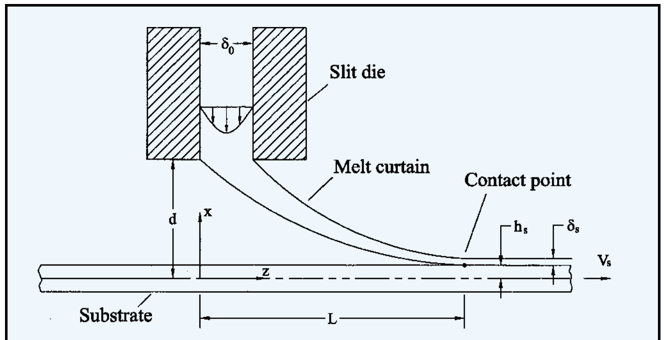

Figure 4—Curtain coating with the die slit perpendicular to the moving substrate.

To obtain the contact convexity, we differentiate equation (45) twice,

$$\left.\frac{\partial^2 x^i}{\partial z^{i2}}\right|_{z^i=L} = \frac{2}{d}\left(\frac{ad}{b_3}\right)^2\left(\frac{b_3}{V_s}\right)^3 \tag{55}$$

Example 2: Calculate the Contact Length and Drawing Force

Given the substrate half thickness $h_s = 5.000 \times 10^{-4}\ m$, die dimensions $\delta_0 = 5.000 \times 10^{-3}\ m$, distance $d = 2.500 \times 10^{-3}\ m$, coating thickness $\delta_s = 1.000 \times 10^{-4}\ m$, coating width $w = 0.9000\ m$, substrate speed $V_s = 2.00\ m/s$ and melt viscosity $\mu = 1.00 \times 10^4$ Pa·s, calculate the contact length L and the corresponding drawing force F_z.

Combining equations (50) and (51) gives $b_3 = h_s V_s/d = 0.4000\ m/s$. Equation (48) gives $V_0 = \delta_s V_s/\delta_0 = 4.000 \times 10^{-2}\ m/s$. From equation (49), one then obtains the drawing force $F_z = 4\mu w \delta_s V_0/d = 57.60\ N$. Finally, from equation (50) or (51), one gets the contact length $L = 0.1000\ m$.

Incompressible Noll Simple Fluid

Following the same arguments as in the parallel slit analysis, we find that when the slit is perpendicular to the substrate,

$$(S_{xx} - S_{zz}) \sim a = \frac{V_0}{d} \tag{56}$$

but the proportionality factor depends upon the particular type of Noll simple fluid, and this factor must therefore be measured in an extensional flow.

CONCLUSION

Table 3 summarizes the results and helps us compare the parallel and perpendicular slit curtain coating. We have shown that, for the parallel curtain coating, an indeterminacy arises. Thus, when the volumetric flow rate of the melt and die dimensions are specified, the drawing force must be controlled to obtain a stable contact length. For the perpendicular curtain coating, this indeterminacy is removed and one can specify the substrate speed or the drawing force to obtain the stable contact length.

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