# Angular Plane Curtain Coating by Drawdown of Extruded Polymer

Fan Ding and A. Jeffrey Giacomin†—University of Wisconsin*
John C. Slattery—Texas A&M University**

*An approximate analytic solution for an angular plane curtain coating of a highly viscous Newtonian fluid is developed. We obtain expressions for the melt curtain shape, coating thickness, contact length, contact pressure, drawing force, apparent contact angle, and contact convexity. Specially constructed examples enable practitioners to apply the results without advanced training in fluid mechanics. We identify a process indeterminacy that arises in curtain coating employing a parallel slit. We show that, with a rapidly converging extrusion slit, this indeterminacy vanishes. Thus, a unique solution for the contact length is always obtained. The extension to a viscoelastic fluid is also briefly considered.*

## INTRODUCTION

In sheet coating, a protective or functional polymeric liquid film is applied to a solid substrate, which is then dried or cured to solidify. This is used to coat paper, packaging, photographic films, labels, tapes, and paintings. Film thicknesses typically exceed one micron.[1]

The many ways to coat sheet have been capably reviewed by Booth,[2] Middleman,[3] Kistler,[4] Kistler and Scriven,[5] Pearson,[6] Ruschak,[7] Satas,[8] Tanner,[9] and Kistler and Schweizer.[1]

In curtain coating, the polymer melt (like polyolefins) is extruded through a slit die and drawn down onto the moving substrate outside the die. The heart of such processes is the slit die. There are two popular slit orientations in curtain coating: parallel (*Figure* 1) and perpendicular (*Figure* 2) to the moving substrate. When an intermediate extrusion angle is used (*Figure* 3), the process is called angular plane curtain coating.[2] Angular plane coater dies are used to coat weak webs, like thin paper. By weak we mean that they cannot withstand much drawing force.

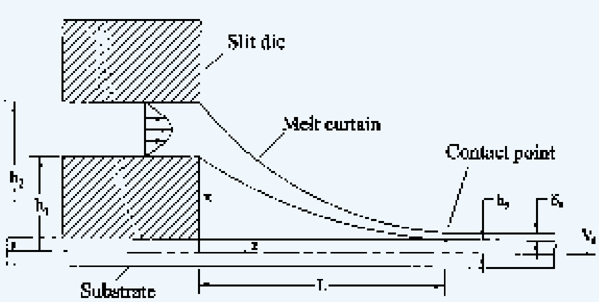


*Figure 1—In parallel curtain coating, the melt curtain is extruded from a slit die and then drawn onto the substrate.*

Finnicum et al.[10] and Benkreira and Cohu[11] studied angular curtain coating, but focused on coating by low viscosity fluids, where gravity, fluid inertia, and surface tension matter, and where viscous forces do not. Flows in these processes are called inviscid.

When coating with highly viscous liquids like molten plastics, viscous effects dominate. This paper provides an approximate analytic solution for angular curtain coating of highly viscous polymer melts and gives expressions for the shape of the freely drawn melt curtain, coating thickness, contact length, apparent contact angle, contact convexity, contact pressure, and drawing force. We determine these quantities, given the volumetric flow rate of the polymer melt, die dimensions, coating width, and substrate speed. We further explore the roles played by the drawing force and extrusion angle. Here we are specifically concerned with the case without any pressure difference across the film thickness.

## SIMPLICATIONS

(1) The polymer melt is incompressible. We will examine two cases: a Newtonian fluid and a Noll simple fluid.[12-15]

†Corresponding author.
*Mechanical Engineering Department and Rheology Research Center, Madison, WI 53706-1572.
**Chemical Engineering Department, College Station, TX 77843-3141.

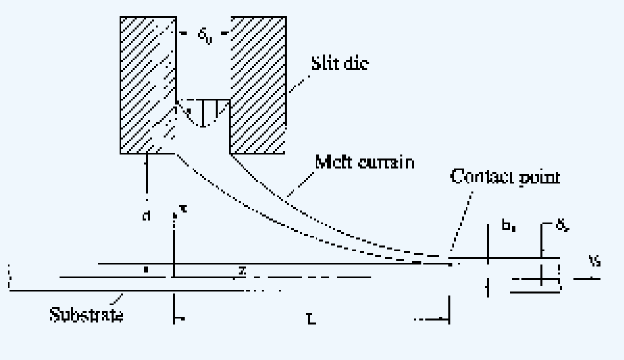


*Figure 2—Curtain coating with the die slit perpendicular to the moving substrate.*

(2) For commercial high speed melt coating, the *Reynolds number* is

$$\mathrm{Re} \equiv \frac{\rho \delta_s V_s}{\mu} << 1 \tag{1}$$

where $\delta_s$ is the coating thickness, $V_s$ is the substrate speed, and $\mu$ is the melt viscosity. Thus fluid inertia is negligible.

(3) For commercial high speed melt coating, the *Poiseuille number* is

$$\mathrm{Ps} \equiv \frac{\rho g d L}{\mu V_s} << 1 \tag{2}$$

where $g$ is the acceleration due to gravity, $d$ is the distance between the substrate midplane and the die exit (*Figure* 3), and $L$ is the contact length. Thus, gravity can be ignored.

(4) Neglect exit effects immediately downstream of the die, such as die swell, which matter in viscoelastic fluids. Hence, the position $z = 0$ is chosen at the die exit.

(5) Assume that the extrusion speed $V_0$ is constant across the die orifice.

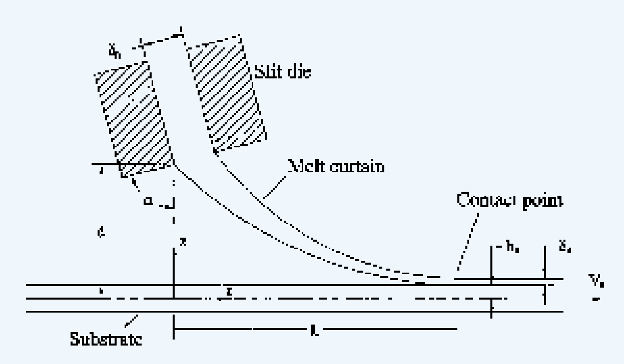


*Figure 3—Angular plane curtain coating with the die slit oriented at an acute extrusion angle to the moving substrate.*

(6) For commercial high speed melt coating, the capillary number is

$$\mathrm{Ca} \equiv \frac{\mu V_0}{\sigma} >> 1 \tag{3}$$

where $\sigma$ is the surface tension. Hence, surface tension effects can be neglected with respect to the viscous effects.

(7) Since the contact length $L$ is well below the coating width $w$, we can further neglect the melt motion in the transverse direction, thus $v_y = 0$ everywhere in the curtain.

(8) Assume that the pressure on both sides of the melt curtain is atmospheric.

## ANALYSIS

*Figure* 3 depicts our rectangular coordinates for the angular curtain where the slit is oriented at any acute extrusion angle. The extrusion speed $V_0$ at the die exit is

$$V_0 = \frac{Q}{w\delta_0} \tag{4}$$

where $Q$ is the volumetric flow rate of polymer melt, and $\delta_0$ is the slit gap.

We will seek a solution in the form of an extensional flow, a subclass of potential flows such that in rectangular coordinates, the velocity potential function is

$$\Phi = -\frac{1}{2}\left(a_1 x^2 + a_2 y^2 + a_3 z^2\right) - b_1 x - b_2 y - b_3 z \tag{5}$$

and the corresponding velocity distribution is $v = \nabla\Phi$. Thus,

$$\begin{aligned} v_x &= a_1 x + b_1 \\ v_y &= a_2 y + b_2 \\ v_z &= a_3 z + b_3 \end{aligned} \tag{6}$$

Continuity (the mass balance) requires

$$a_1 + a_2 + a_3 = 0 \tag{7}$$

By simplification 7, $v_y = 0$, thus

$$a_2 = b_2 = 0 \tag{8}$$

In view of equations (7) and (8)

$$a_1 = -a_3 \tag{9}$$

Here we introduce

$$a \equiv a_3 \tag{10}$$

which we call the *curtain shape factor*. From the boundary condition $v_x = 0$ at $x = 0$, we get

$$b_1 = 0 \tag{11}$$

In summary,

$$v_x = -ax$$
$$v_y = 0$$
$$v_z = az + b_3 \tag{12}$$

We now seek the pathline. Integrating equation (12) with initial conditions at $t = 0$,

$$x = x_0,\ y = y_0,\ z = z_0 \tag{13}$$

gives

$$\frac{x}{x_0} = e^{-at}$$
$$\frac{y}{y_0} = 1 \tag{14}$$
$$\frac{az + b_3}{az_0 + b_3} = e^{at}$$

Eliminating $t$ gives the shape of both curtain surfaces,

$$\frac{x}{x_0} = \left(\frac{az + b_3}{az_0 + b_3}\right)^{-1} \tag{15}$$

For the inner curtain surface $x_o^i = d,\ z_o^i = 0$; for the outer surface, $x_0^o = d + \delta_0 \cos\alpha,\ z_0^o = \delta_0 \sin\alpha$. In this paper, the superscripts *i* and *o* denote the quantities on the inner and outer curtain surfaces, respectively. *Figure* 4 represents the shape of both curtain surfaces from equation (15).

Since the curtain is thin, from equation (15), we obtain the components of unit normals on both curtain surfaces.

$$\xi_x^o \approx -\xi_x^i = \left[1 + \left(\frac{ad}{b_3}\right)^2 \left(\frac{x^i}{d}\right)^4\right]^{-1/2}$$
$$\xi_y^o = \xi_y^i = 0 \tag{16}$$
$$\xi_z^o \approx -\xi_z^i = \left(\frac{ad}{b_3}\right)\left(\frac{x^i}{d}\right)^2 \left[1 + \left(\frac{ad}{b_3}\right)^2 \left(\frac{x^i}{d}\right)^4\right]^{-1/2}$$

which will be required shortly.

## Incompressible Newtonian Fluid

From equation (12), for an incompressible Newtonian fluid, the equation of motion (recall that fluid inertia is neglected) reduces to

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \tag{17}$$

which means that, $p$ = constant in the curtain.

On both the inner and outer curtain surfaces, the $x$-component of the momentum balance requires that

$$(T_{xx} + p_0)\,\xi_x = 0 \tag{18}$$

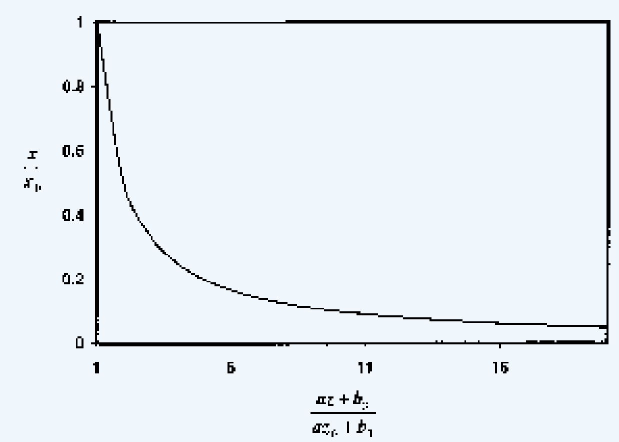


*Figure 4—The shape of inner and outer film surfaces in angular curtain coating, given by equation (15).*

or that

$$(-p + 2\,\mu a + p_0)\,\xi_x = 0 \tag{19}$$

In agreement with equation (17),

$$p = p_0 + 2\,\mu a \tag{20}$$

The $y$-component of the momentum balance is satisfied on both curtain surfaces. The $z$-component, on both surfaces, takes the form

$$(-p - 2\,\mu a + p_0)\,\xi_z = 0 \tag{21}$$

Equations (18) to (21) are valid only when $\delta_x >> \delta_z$, or inserting equation (16), when

$$\frac{ad}{b_3} << 1 \tag{22}$$

where $ad$ is the magnitude of $v_x$ at the die exit.

The $z$-component of the force that the substrate exerts on the liquid coating (beyond the opposing force of atmospheric pressure) is,

$$\begin{aligned} F_z &= \int_0^w \int_{h_s}^{h_s+\delta_s} (t_z - p_0)\,dxdy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (-T_{zz} - p_0)\,dxdy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (p + 2\,\mu a - p_0)\,dxdy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} (4\mu a)\,dxdy \\ &= 4\mu a w \delta_s \end{aligned} \tag{23}$$

As expected, $F_z$ is positive. From equation (23),

$$a = \frac{F_z}{4\mu w \delta_s} \tag{24}$$

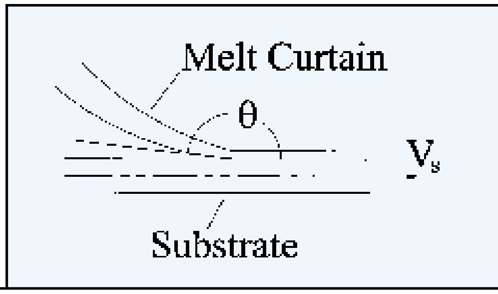


Figure 5—Apparent contact angle in curtain coating.

Assuming constant density, the overall mass balance requires a coating thickness of

$$\delta_s = \frac{Q}{V_s w} = V_0 \delta_0 / V_s \tag{25}$$

Evaluating equation (15) at the contact point ($x^i = h_s$, $z^i = L$) gives

$$L = \frac{b_3}{a}\left[\frac{d}{h_s} - 1\right] \tag{26}$$

Finally, from the $z$-component of velocity in equation (12)

$$V_s = aL + b_3 \tag{27}$$

From equations (22) and (27), we find that these solutions apply when:

$$\frac{1}{\frac{V_s}{ad} - \frac{L}{d}} << 1 \tag{28}$$

Combining equations (24) and (26) and then introducing the dimensionless contact length

$$L^* \equiv \frac{L}{h_s} \tag{29}$$

and the dimensionless drawing force

$$F_z^* \equiv \frac{F_z}{4\,\mu b_3 w} \tag{30}$$

gives

$$L^* F_z^* = \delta_s^* \left(d^* - 1\right) \tag{31}$$

where $\delta_s^* \equiv \delta_s / h_s$ is the dimensionless coating thickness, and $d^* \equiv d/h_s$ is the dimensionless distance between the slit and substrate.

We now seek the apparent contact angle defined as (*Figure* 5)

$$\theta_c = \pi - \arctan\left(\left.\frac{\partial x^i}{\partial z^i}\right|_{z^i = L}\right) \tag{32}$$

Differentiating equation (15) and combining with equations (29) and (30) yields

$$\theta_c = \pi - \arctan\left(\frac{F_z^*}{\delta_s^* d^*}\right) \tag{33}$$

In practice, when the apparent contact angle approaches 180°, air is entrained between the polymer coating and substrate, thus good adhesion is lost. When the apparent contact angle is large, we believe that air entrainment may still be avoided if the contact convexity is high. We obtain the contact convexity by differentiating equation (15) twice,

$$\left.\frac{\partial^2 x^i}{\partial z^{i2}}\right|_{z^i = L} = \frac{2}{d}\left(\frac{ad}{b_3}\right)^2 \left(\frac{b_3}{V_s}\right)^3 \tag{34}$$

## Effect of Extrusion Angle and Process Indeterminacy

For small extrusion angles, $b_3$ ($v_z$ near the slit) is

$$b_3 \approx V_0 \cos\alpha \tag{35}$$

Given the drawing force $F_z$, volumetric flow rate of the polymer melt $Q$ [thus $V_0$ from equation (4)], coating width $w$, substrate half thickness $h_s$, die dimension $\delta_0$ and distance $d$, one can combine equations (24) through (27) to determine the final coating thickness $\delta_s$, substrate speed $V_s$, curtain shape factor $a$, and contact length $L$. Interestingly, if one specifies $V_s$, rather than $F_z$, the problem may become indeterminate. By indeterminate we mean that equations (26) and (27) result in two independent equations for the product $aL$.

For large extrusion angles (near 90°), $v_x$ near the slit is

$$v_x = -ad \approx -V_0 \sin\alpha \tag{36}$$

thus,

$$a \approx \frac{V_0 \sin\alpha}{d} \tag{37}$$

Given the drawing force $F_z$ (or substrate speed $V_s$), volumetric flow rate of the polymer melt $Q$ [thus $V_0$ from equation (4)], coating width $w$, substrate half thickness $h_s$, die dimension $\delta_0$ and distance $d$, one can combine equations (24) through (27) to determine the final coating thickness $\delta_s$, substrate speed $V_s$ (or $F_z$), $b_3$ ($v_z$ near the slit) and contact length $L$. Thus, the process indeterminacy that arose in small extrusion angle curtain coating is eliminated.

**Example 1: Contact Length and Drawing Force Calculation:** Given the substrate half thickness, $h_s = 5.000 \times$

$10^{-4}$ *m*, die gap $\delta_0 = 5.000 \times 10^{-3}$ *m*, die proximity $d = 2.500 \times 10^{-3}$ *m*, coating thickness $\delta_s = 1.000 \times 10^{-4}$ *m*, coating width $w = 0.9000$ *m*, substrate speed $V_s = 2.000$ *m/s* and melt viscosity $\mu = 1.000 \times 10^4$ *Pa·s*, calculate the contact lengths $L$ and drawing forces $F_z$ for several extrusion angles $\alpha$.

Combining equations (26) and (27), we have $b_3 = h_s V_s / d = 0.4000$ *m/s*. Thus from equation (27), $aL = V_s - b_3 = 1.600$ *m/s*. A mass balance on the melt curtain, equation (25), gives $V_0 = V_w \delta_s / \delta_0 = 4.000 \times 10^{-2}$ *m/s*. *Table* 1 lists the results for four extrusion angles.

From *Table* 1, the contact length decreases with the extrusion angle, whereas the drawing force increases.

**Example 2: Die Design**: Given the substrate half thickness $h_s = 5.500 \times 10^{-4}$ *m*, die gap $\delta_0 = 6.000 \times 10^{-3}$ *m*, die proximity $d = 2.500 \times 10^{-3}$ *m*, coating thickness $\delta_s = 1.500 \times 10^{-4}$ *m*, coating width $w = 0.9000$ *m*, substrate speed $V_s = 1.000$ *m/s* and melt viscosity $\mu = 1.000 \times 10^4$ *Pa·s*, find the extrusion angle $\alpha$ which gives a contact length $L$ of $8.000 \times 10^{-2}$ *m* and compute the corresponding drawing force $F_z$.

Combining equations (26) and (27), we have $b_3 = h_s V_s / d = 0.2200$ *m/s*. Thus, from equation (27), $a = (V_s - b_3)/L = 9.750\ s^{-1}$. Equation (25) gives $V_0 = V_w \delta_s / \delta_0 = 2.500 \times 10^{-2}$ *m/s*. From equation (36), we have $\sin\alpha \approx \frac{ad}{V_0} = 0.9750$, thus the extrusion angle is $\alpha = 77.16°$. The corresponding drawing force, from equation (23), is then $F_z = 52.65$ N.

### Incompressible Noll Simple Fluid

Nearly all viscoelastic fluid behavior falls into a general class of rheological models called the Noll simple fluid. For an incompressible Noll simple fluid, the extra stress tensor $S$ is a functional of the history of either the right relative Cauchy-Green tensor,[12,14] or simply the relative deformation gradient.[13]

$$S \equiv p\delta + T = \underset{t'=-\infty}{\overset{t'=t}{f}}\left(C(t')\right) \tag{38}$$

Given the particle paths in equation (15), both the right relative Cauchy-Green strain tensor and the relative deformation gradient are independent of position.[16,17] This means that the components of $S$ for a Noll simple fluid are also independent of position[16,17] except for the dependence on temperature of the physical parameters used to describe a particular member of this class of fluids.

**Table 1—Contact Lengths and Drawing Forces for Several Extrusion Angles**

| α (°) | a (s⁻¹) | L (m) | $F_z$ (N) |
|---|---|---|---|
| | $a \approx \frac{V_0 \sin\alpha}{d}$ | $L = \frac{V_s - b_3}{a}$ | $F_z = 4\mu a w \delta_s$ |
| 60 | 13.86 | 0.1155 | 49.90 |
| 70 | 15.04 | 0.1064 | 54.14 |
| 80 | 15.76 | 0.1015 | 56.74 |
| 90 | 16.00 | 0.1000 | 57.60 |

The equation of motion reduces to equation (17) with the same arguments used in the Newtonian analysis. We conclude that

$$p = p_0 + S_{xx} \tag{39}$$

and that three components of the momentum balance are satisfied. Hence,

$$\begin{aligned} F_z &= \int_0^w \int_{h_s}^{h_s+\delta_s} \left(p - S_{zz} - p_0\right) dxdy \\ &= \int_0^w \int_{h_s}^{h_s+\delta_s} \left(S_{xx} - S_{zz}\right) dxdy \\ &= \left(S_{xx} - S_{zz}\right) w\delta_s \end{aligned} \tag{40}$$

In view of equation (23)

$$(S_{xx} - S_{zz}) \sim a \tag{41}$$

but the proportionality factor depends upon the particular type of Noll simple fluid, and this factor must therefore be measured in an extensional flow. Thus, for viscoelastic fluids, the parameter $a$ must be viewed as unknown. Equations (25) through (27) still apply, but they are not sufficient to specify $L$ and $V_s$.

## CONCLUSION

An approximate analytic solution for an angular plane curtain coating with any acute extrusion angle is presented. Expressions have been derived for the melt curtain shape, coating thickness, contact length, contact pressure, drawing force, apparent contact angle, and contact convexity. These results can assist in coating die design. Specially constructed examples enable practitioners to apply the results without advanced training in fluid mechanics. A process indeterminacy arises in curtain coating with the slit parallel to the moving substrate. With a rapidly converging extrusion slit, this process indeterminacy vanishes.

## ACKNOWLEDGMENT

The financial support of the Kimberly-Clark Corp., the DuPont Co., and the 3M Co. is gratefully acknowledged.

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